OPTIMIZATION OF CHANNEL PROFIT FOR DETERIORATING ITEMS UNDER PERMISSIBLE DELAY IN PAYMENTS WHEN END DEMAND IS PRICE SENSITIVE

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ABSTRACT
This paper considers the seller - buyer channel for deteriorating items in which the demand rate is expressed as a function of price and the seller may offer the credit period to the buyer. We determine the optimal cycle length and unit-selling price charged by the buyer and optimal per unit price and the length of credit period offered by the seller, which jointly maximizes the channel profit. The numerical solution of the model is obtained and the sensitivity of the parameters involved in the model is also examined.

Keywords: Deterioration, Permissible delay in payments, Channel coordination.

MSC 62K25

RESUMEN:
Este trabajo considera el canal vendedor-comprado para productos deteriorables para los cuales la tasa de demanda se expresa como una función del precio y el vendedor ofrece un periodo de crédito al comprador. Nosotros determinamos el largo optimo del ciclo y el precio de venta por unidad cargado al comprador y optimo por unidad de precio y el largo del periodo de crédito ofrecido por el vendedor. que maximizan simultáneamente el canal de ganancia. La solución numérica del modelo es obtenida y la sensibilidad del parámetro envuelto en el modelo es examinada también

1. INTRODUCTION

One of the important problems faced in inventory management that how to control and maintains the inventories of deteriorating items. Food items, pharmaceuticals, chemicals and blood are a few examples of such items. The decrease in utility or loss for an inventory of goods subject to deterioration is usually a function of the total amount of inventory on hand. The analysis of decaying inventory problem began with *Ghare* and *Schrader* [1963], who proposed an inventory model having a constant rate of deterioration and constant rate of demand over a finite-planning horizon. Now-a-days deterioration is a well-established fact in literature *Chang and Dye* [2001]**,** *Chung* [2000], *Covert and Phillip* [1973], *Hwang and Shinn* [1997], *Rafaat* [1991] *Shah and Jaiswal* [1977] and its effect cannot be ignored as it may yield misleading results.

Further, the classical EOQ model work under the assumption that the buyer must pay to the supplier immediately after receiving the goods. But this assumption is not true in most of the cases, as most of the supplier offer certain fixed period (credit period) for settling the amount owed to them for the items supplied. During the credit period, the seller burdens the capital opportunity cost for the goods sold to the buyer. So the buyer just bears the physical holding cost before he pays for the goods at the end of credit period. The main purpose for the seller in providing credit period to the buyer is the stimulation of the end demand of the goods. It will be

economical for the seller if the increased sales are sufficient to compensate the opportunity cost incurred. And the buyer can take advantage of a credit period that reduces his costs and increases his profit.

Over years, a number of researches have been published which dealt with the economic order quantity problems under conditions of permissible delay in payments described above. About the relationship between credit period and demand, *Mehta* [1968] implicitly stated the supplier usually expected the profit to increase by the increment of sales volume and compensate the capital losses incurred during the credit period. Also, *Fewings* [1992] pointed out that the advantage of providing credit period was substantial in terms of influence on the purchasing of the buyer and marketing decisions. In the past, *Chapman et al.* [1985] examined the effect of the credit period on the optimal inventory policy. The methods in calculating inventory cost could be divided into two ways. The first was the average cost approach. The capital cost was calculated with the concepts of credit surplus, balance and deficit as introduced by *Haley and Higgins* [1973]. They stated a model in which payment was made at the end of a fixed period of time after the order was received and where the borrowing and the lending rates of the company were different. When the credit period was greater or equal to the cycle time *(T),* only credit balances occurred. When the credit period was less than *T* credit balances and credit deficits occur. A credit surplus arose when credit balances exceeded inventory investment; conversely a credit deficit arose when inventory investment exceeded credit balances. Also, *Chapman et al. [1985]* and *Goyal* [1985] utilized similar concept to calculate the holding cost.

Another way was the discounted cash flow (DCF) approach, which used by other authors has shown that the order quantity was an increasing function of the length of delay in the payment allowed by suppliers. *Chung* [1989] presented the DCF approach for the analysis of the optimal inventory policy in the presence of the trade credit. *Chung and Huang* [2000] further characterized and determined the behavior and optimal inventory cycle time of the present value function of all future cash outflows. *Chung* [1989] solved the problem with a near-optimal solution whereas *Chung* [1999] could determine an optimal solution.

The related literatures about credit period can also be divided by the categories of the buyer's, the seller's and the channel's points of view. From buyer's point of view, researches assumed that the seller offered a specified credit period to the buyer, and investigated the related subjects. *Jaggi and Aggarwal* [1994] developed an inventory model for obtaining the optimal order size of deteriorating items in the presence of trade credit using the DCF approach. *Jamal et al.* [2000] developed a buyer's model for optimal strategy for payment time.

Goyal [1985] developed mathematical models for determining the economic order quantity under the conditions of permissible delay in payments from the perspective of buyer. Chung [1998] simplified the search for the optimal solution to the problem. *Teng* [2002] then amended *Goyal's* model by considering the difference between unit price and unit cost.

Shinn [1997] dealt with the problem of determining the buyer's optimal price and lot size simultaneously under the condition of permissible delay in payments. Besides, Shinn *et al.* [1996] considered similar problem with the condition that the freight cost had a quantity discount. *Hwang and Shinn* [1997] dealt with the same problem for an exponentially deteriorating product. Then, *Shinn and Hwang* [2003] discussed the condition of order-size-dependent delay in payments.
From seller's point of view, *Kin et al.* [1995] developed an optimal credit policy to increase seller's profit with price-dependent demand functions. It dealt with the problem of determining an optimal length of credit period from the perspective of supplier. They assumed that a buyer jointly determined the retail price and order size to maximize profit when he purchased a product for which the supplier offered a credit period. Two common demand functions were considered: the constant price elasticity function and the linear demand function.

From channel's point of view, there are not many researches that dealt with this topic under conditions of credit period. Recently, *Abad and Jaggi* [2003] dealt with the problem of determining

the optimal credit period from the channel perspective. They provided procedures for the seller's and the buyer's policies under non-cooperative as well as cooperative relationship. In their research, they assumed the seller's capital opportunity cost to be a linear and increasing function of the credit period, and they utilized short-term capital gain and short-term capital gain and short-term capital cost to calculate the buyer's inventory cost and provided a procedure for characterizing Pareto efficient solutions in the cooperative structure, and discussed the influence of different credit period on buyer's and seller's profits.

In this paper we considers the seller-buyer channel for deteriorating items in which the demand rate is expressed as a constant price elasticity function and the seller may offer the credit period to the buyer to stimulate demand. We determine the optimal cycle length and unit selling price for the buyer and the optimal selling price and the length of the credit period for the seller, which jointly maximizes the channel profit.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions and notations are used in the paper:

Assumptions:

1. The demand rate for the product is elastic i.e. $e \geq 1$.
2. Replenishment rate is infinite.
3. Shortages are not allowed.
4. A constant fraction of the on-hand inventory deteriorates per unit time.
5. There is no repair or replenishment of the deteriorated items during the inventory cycle.
6. Deterioration effects only the buyer and not the seller i.e. loss due to deterioration of items is covered by the buyer
7. Planning horizon is infinite.
8. $I_s = a+bM$, $a>0$ and $b>0$ i.e. seller's opportunity cost is a linearly increasing function of M.
9. The seller's follow a-lot-for-lot strategy. Thus the seller does not incur carrying cost associated with the lot size Q.
10. Seller provide credit period to the buyer for a fixed period, however there are no cash discounts for settling the account early. At the end of credit period, buyer must settle the amount. Thus as shown in the *figure1* during the credit period, buyer has credit balance and enjoys short term capital gain at the rate I_p. After the credit period is over, the account is settled and the buyer has credit deficit because of financing of the inventory at the rate I_c. Moreover, $I_p = I_c$ (*Haley and Higgins*).

Notations:

A_b : Buyer's ordering cost per order.
A_s : Seller's ordering cost per order.
I_e : Interest that can be earned per rupee in a year.
I_p : Interest paid per rupee investment in stock in a year.
I_b : Inventory carrying charge per year excluding the cost of financing.
$I = I_p + I_b$
$D(p)$: Kp^{-e} : Annual demand rate as the function of retail price. For notational simplicity $D(p)$ and D will be used interchangeably.
e : Index of price elasticity. $e \geq 1$.
P : Buyer's retail price(buyer's decision variable).
Q : Buyer's lot size.
T : Cycle time.
I_s : Seller's opportunity cost of capital.
c : Seller's unit purchase cost.
v : Price charged by the seller to the buyer (seller's decision variable).
M : Credit period (seller's decision variable).

θ : Constant rate of deterioration ($0 \le \theta \le 1$).

3. MATHEMATICAL FORMULATION

Let $I(t)$ be the inventory level at any time t, ($0 \le t \le T$). Depletion due to deterioration and demand will occur simultaneously. The differential equation describing the instantaneous state of $I(t)$ over $(0,T)$ is given by:

$$\frac{dI(t)}{dt} + \theta I(t) = -D \qquad 0 \le t \le T \tag{1}$$

Solution to the equation (1) (using the boundary condition $I(t) = 0$ at $t = T$) is given by

$$I(t) = \frac{D}{\theta}(e^{\theta(T-t)} - 1) \tag{2}$$

Also at $t = 0$ $I(t) = Q$

$$\Rightarrow Q = \frac{D}{\theta}(e^{\theta T} - 1) \tag{3}$$

Total demand during one cycle is DT.

Total no. of units deteriorated in one cycle is given by

$(Q-DT)$ $= \frac{D}{\theta}(e^{\theta T} - 1) - DT$.

4. THE BUYER'S PROBLEM

The buyer's objective is to set the retail price and the cycle length in such a way that his net profit is maximized. Now based on the length of the credit period offered by the seller, two cases arise, namely $M \le T$ & $M \ge T$.

We first consider the case1 when $M \le T$. In this case the buyer starts getting the sales revenues and earns interest on average sales revenue for the time period [0, M]. At M accounts are settled, if the stock still remains, finances are to be arranged to make the payments to the supplier.

The net profit function consists of the following elements.

1. Sales revenue per cycle = pDT
2. Purchasing cost per cycle = vQ = $v\frac{D}{\theta}\left[e^{\theta T} - 1\right]$ using equation (3).
3. Ordering cost per cycle = A_b
4. Inventory carrying cost (excluding the cost of financing) per cycle= $I_b v\int_0^T I(t)dt$

$$= I_b . v\int_0^T \frac{D}{\theta}(e^{\theta(T-t)} - 1)dt = \frac{\mathrm{I_b.v.}D}{\theta^2}(e^{\theta T} - 1 - \theta T)$$

5. Interest earned per cycle during the time span [0, M] is = $I_e p \int_0^M D t dt$

$$= \frac{I_e.p.DM^2}{2}$$

6. Interest payable per cycle during the time span [M, T]= $I_{p.} v \int_M^T I(t) dt$

$$= \frac{I_p.v.D}{\theta^2}\left[e^{\theta(T-M)} - 1 - \theta(T-M)\right]$$

Therefore, the profit per cycle π_{b1} (p, T) can be expressed as

π_{b1} (p, T) = sales revenue - purchase cost – ordering cost - inventory carrying cost + interest earned – interest paid.

$$= pDT - \frac{vD}{\theta}(e^{\theta T} - 1) - A_b - \frac{I_b vD}{\theta^2}(e^{\theta T} - 1 - \theta T) + \frac{I_e pDM^2}{2} - \frac{I_p vD}{\theta^2}\left[e^{\theta(T-M)} - 1 - \theta(T-M)\right]$$

Hence, the profit per unit time is given by

$$Kp^{-e}\left[p - \frac{v}{\theta T}(e^{\theta T} - 1) - \frac{I_b v}{T\theta^2}(e^{\theta T} - 1 - \theta T) + \frac{I_e pM^2}{2T} - \frac{I_p v}{\theta^2 T}\left[e^{\theta(T-M)} - 1 - \theta(T-M)\right]\right] - \frac{A_b}{T}$$

The problem is to find an optimum retail price *p** and an optimal replenishment cycle time *T** which maximizes π_{b1} (p, T). Once *p** and *T** are found, an optimal *Q** can be obtained from equation (3). Although the objective function is differentiable, the resulting equation is mathematically intractable i.e. it is difficult to express the optimal solution in explicit form. Thus, approximately by using a truncated Taylor series expansion for the exponential term, the model is solved.

$e^{\theta T} = 1 + \theta T + \frac{\theta^2 T^2}{2}$ which is a valid approximation for θT <1.

With the above approximation the annual net profit function can be rewritten as

$$\pi_1 (p,T) = Kp^{-e}\left[p - v(1 + \frac{\theta T}{2}) - \frac{I_b vT}{2} + \frac{I_e pM^2}{2T} - \frac{I_p v(T-M)^2}{2T}\right] - \frac{A_b}{T} \tag{4}$$

Expression (4) is derived assuming $M \le T$. For the case2 where $M \ge T$, the first four components of the profit function remain same. The sixth cost component does not exist for $M \ge T$. The interest eared during the time span [0, *M*] is

5. $I_e p\int_0^T Dtdt + I_e pDT(M-T) = I_e pDT^2 + I_e pDT(M-T) = I_e pDMT - \frac{I_e pDT^2}{2}$

In this case profit for the buyer is given by
sales revenue - purchase cost - ordering cost - inventory carrying cost + interest earned

$$= pDT - \frac{vD}{\theta}(e^{\theta T} - 1) - A_b - \frac{I_b vD}{\theta^2}(e^{\theta T} - 1 - \theta T) + I_e pDMT - \frac{I_e pDT^2}{2}$$

and the profit per unit time is given by

$$\pi_{b2}(p,T) = pD - \frac{vD}{\theta T}(e^{\theta T} - 1) - \frac{A_b}{T} - \frac{I_b vD}{T\theta^2}(e^{\theta T} - 1 - \theta T) + I_e pDM - \frac{I_e pDT}{2}$$

$$= Kp^{-e}\left[p - \frac{v}{\theta T}(e^{\theta T} - 1) - \frac{I_b v}{T\theta^2}(e^{\theta T} - 1 - \theta T) + I_e pM - \frac{I_e pT}{2}\right] - \frac{A_b}{T}$$

Using the same approximation as in case 1:

$e^{\theta T} = 1 + \theta T + \dfrac{\theta^2 T^2}{2}$, which is valid for $\theta T < 1$

$$\pi_2(p,T) = Kp^{-e}\left[p - v(1 + \frac{\theta T}{2}) - \frac{I_b vT}{2} + I_e pM - \frac{I_e pT}{2}\right] - \frac{A_b}{T} \qquad (5)$$

Now, we take the first and second derivative of π_{b1} (p, T) and π_{b2} (p, T) w.r.t. T, for a fixed p, which gives us T_1 and T_2 respectively.

$$T_1 = \sqrt{\frac{2A_b + M^2 Kp^{-e}(I_p v - I_e p)}{Kp^{-e} v(\theta + I_p + I_b)}} \qquad (6)$$

$$T_2 = \sqrt{\frac{2A_b}{Kp^{-e}(v\theta + I_e p + I_b v)}} \qquad (7)$$

It has been verified that sufficient condition holds good provided $I_p v > I_e p$.

Therefore, for a fixed p both the functions are concave function of T.

Since demand is a function of p, so T can be represented by a real value function of p i.e.

$T_1 = T_1(p)$ and $T_2 = T_2(p)$.

Now substituting (6) and (7) in (4) and (5) resp. and after simplification

$$\pi_1(p) = Kp^{-e}\left(p - v + I_p vM\right) - \sqrt{\left\{2A_b + M^2 Kp^{-e}(I_p v - I_e p)\right\} Kp^{-e} v(\theta + I_b + I_p)}$$

$$\pi_2(p) = Kp^{-e}\left(p - v + I_e pM\right) - \sqrt{2A_b Kp^{-e}(v\theta + vI_b + pI_e)}$$

$$\frac{\partial \pi_1(p)}{\partial p} = 0 \quad and \quad \frac{\partial \pi_2(p)}{\partial p} = 0$$

This gives us the optimal values of p_1 and p_2.

In order to find a closed form expression, we assume $I_p = I_e$, $I = I_p + I_b$ (*Halley and Higgins*) and $p = v$, after simplification we get

$$\pi_b(p,T) = Kp^{-e}\left[p - v\left(1 + \frac{\theta T}{2} + \frac{IT}{2} - I_p M\right)\right] - \frac{A_b}{T} \tag{8}$$

For necessary and sufficient conditions with respect to *T*,

$$\frac{\partial \pi_b}{\partial T} = -vKp^{-e}\frac{(\theta + I)}{2} + \frac{A_b}{T^2} \tag{9}$$

$$\frac{\partial^2 \pi_b}{\partial T^2} = -\frac{2A_b}{T^3} < 0 \tag{10}$$

For a fixed *p*, $\pi_b(p,T)$ is a concave function of *T*, and there exist a unique value of *T*, which maximizes $\pi_b(p,T)$ which is given by:

$$T^*(p) = \sqrt{\frac{2A_b}{Kp^{-e}v(\theta + I)}} \tag{11}$$

Note since demand is a function of price (*p*), *T* can be represented by a real valued function of *p*; that is *T* = *T*(*p*). Substituting (11) in (8), we have

$\pi_b(p, T^*(p)) = \pi_1(p) =$

$$Kp^{-e}\left[p - v\left(1 + \frac{(\theta + I)}{2}\sqrt{\frac{2A_b}{vKp^{-e}(\theta + I)}} - I_p M\right)\right] - \left[\frac{A_b}{\sqrt{2A_b \big/ Kp^{-e}v(\theta + I)}}\right]$$

$$\pi_1(p) = Kp^{-e}\left[p - v(1 - I_p M)\right] - \sqrt{2A_b vKp^{-e}(\theta + I)} \tag{12}$$

$\dfrac{\partial \pi_1(p)}{\partial p} = 0$ implies

$$p = \left(\frac{e}{e-1}\right)v(1 - I_p M + \frac{(\theta + I)}{2}T^*(p)) \tag{13}$$

In the above formulation, we have expressed the buyer's profit as a function of his unit-selling price (*p*). One can use the reverse approach in which the expression (13) is substituted in (8) to eliminate *p* and express final profit as a function of *T*.

5. THE SELLER'S PROBLEM

The buyer's optimal retail price depends upon the credit period (*M*) offered by the seller. Moreover, the demand (D(p)) decreases with an increase in price(*p*). Thus, the credit period has a positive effect on the demand rate and consequently on the buyer's lot size (Q). Given the assumption that the seller follows a lot for a lot policy w.r.t .an individual buyer the seller does not incur any carrying cost. Moreover, the effect of deterioration is only on the buyer and not on the seller. The elements of seller's profit function are:

1. Sales revenue per cycle = $vQ = \dfrac{vD(e^{\theta T}-1)}{\theta}$

2. Purchase cost per cycle = $cQ = \dfrac{cD(e^{\theta T}-1)}{\theta}$

3. Ordering cost per cycle = A_s

4. Opportunity cost per cycle because of offering trade credit = $I_s cMQ$

Thus, the seller's net profit function per cycle is given by
Sales revenue - purchase cost - opportunity cost - ordering cost

$$= \frac{D(e^{\theta T}-1)}{\theta}\left[v - c - I_s cM\right] - A_s$$

and the seller's net profit per unit time $\Pi_s(v,M)$ is given by

$$\Pi_s(v,M) = \frac{D(e^{\theta T}-1)}{T\theta}\left[v - c - I_s cM\right] - \frac{A_s}{T}$$
$$= Kp^{-e}\left(1+\frac{\theta T}{2}\right)\left[v - c - I_s cM\right] - \frac{A_s}{T} \qquad (14)$$

6 **LEADER–FOLLOWER RELATIONSHIP**

In the leader follower structure the seller offers policies v and M to the buyer and the buyer reacts to the seller's policies by maximizing his profit Π_b (*p*, *T)*

The buyer response is given by condition (11) & (13) the seller would like to choose v and M so that his profit is maximized .The seller's problem (P1) thus is

$$\text{Max } \Pi_s(v,M,p,T) = Kp^{-e}\left(1+\frac{\theta T}{2}\right)\left[v - c - I_s cM\right] - \frac{A_s}{T} \qquad (15)$$

subject to: $$p = \left(\frac{e}{e-1}\right)v(1 - I_p M + \frac{(\theta+I)}{2}T \qquad (16)$$

$$T^*(p) = \sqrt{\frac{2A_b}{Kp^{-e}v(\theta+I)}} \qquad (17)$$

$$M \geq 0 \qquad (18)$$

From equation (14),

$$p = \left[\frac{KvT^2(\theta+I)}{2A_b}\right]^{1/e} \qquad (19)$$

From (14) and (17),

$$M = \frac{1}{I_p}\left[1 + \frac{(\theta+I)T}{2} - \frac{e-1}{ev}\left(\frac{KvT^2(\theta+I)}{2A_b}\right)^{1/e}\right] \qquad (20)$$

Table 1

For a fixed Ic (=0.16)						
θ↓	e→	**3**	**2.5**	**2.0**	**1.5**	**1.3**
0.00	*v*	5.4169668	5.8366268	6.8799543	10.213157	14.730294
	p	7.7689500	9.1040147	12.704279	28.054183	58.373591
	T	0.2486574	0.1749429	0.1294474	0.1019422	0.0979200
	M	0.4917909	0.5537966	0.5927526	0.6165602	0.6200656
	D	853.04709	1599.4756	2478.3346	2691.9287	2022.9198
	Q	212.11647	279.81682	320.81390	274.42126	198.08409
	Π_s	606.44820	2236.8829	6130.2729	14491.286	18500.156
	Π_b	2048.2295	5596.0136	15433.721	49954.195	90426.188
0.10	*v*	5.555391	5.922267	6.948294	10.29121	14.83958
	p	8.094375	9.331085	12.92300	28.42768	59.12510
	T	0.224151	0.153743	0.112473	0.088043	0.084443
	M	0.445218	0.524094	0.571432	0.599708	0.603713
	D	754.2408	1503.937	2395.152	2639.052	1989.558
	Q	170.9729	233.0058	270.9101	233.3765	168.7157
	Π_s	409.2929	1953.154	5749.675	14021.80	18021.98
	Π_b	1856.585	5353.170	15120.64	49560.42	90013.08
0.20	*v*	5.700105	6.003312	7.010398	10.36047	14.93595
	p	8.431421	9.544816	13.12123	28.75915	59.78841
	T	0.209317	0.139768	0.101157	0.078757	0.075436
	M	0.400770	0.497433	0.552734	0.585104	0.589616
	D	667.3562	1421.154	2323.331	2593.558	1960.911
	Q	142.654	201.4344	237.4155	205.8769	149.0449
	Π_s	244.4737	1708.720	5416.979	13606.98	17597.58
	Π_b	1684.489	5139.672	14847.05	49217.79	89654.17
0.30	*v*	5.857554	6.082071	7.068454	10.424	15.0239
	p	8.794845	9.751435	13.30599	29.0626	60.3931
	T	0.200118	0.129751	0.092936	0.07199	0.06887
	M	0.356657	0.472767	0.535802	0.57204	0.57703
	D	587.9975	1347.065	2259.258	2553.04	1935.42
	Q	121.2725	178.2295	212.9217	185.798	134.686
	Π_s	103.2533	1492.525	5118.841	13232.1	17212.8
	Π_b	1523.900	4946.045	14600.43	48909.8	89331.8

Substituting *M* and *p* in Π_s and relaxing (16), problem P1 becomes the following unconstrained problem:

$$Max \quad \Pi_s(v,T) = \frac{A_b(2+\theta T)}{vT^2(\theta+I)}\left[v - c - \frac{av}{I_p}\left\{1+\frac{(\theta+I)T}{2} - \frac{e-1}{ev}\left(\frac{KvT^2(\theta+I)}{2A_b}\right)^{1/e}\right\}\right.$$
$$\left. - \frac{bv}{I_p{}^2}\left\{1+\frac{(\theta+I)T}{2} - \frac{e-1}{ev}\left(\frac{KvT^2(\theta+I)}{2A_b}\right)^{1/e}\right\}^2\right] - \frac{A_s}{T} \tag{21}$$

$\Pi_s(v,T)$ is a non-linear objective function. The global maximum of $\Pi_s(v,T)$ can be found by doing an enumerative/grid search. Another approach is to solve problem (P1) for different starting values of (*v, T*) to identify local minima, if they exist.

7. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

We illustrate formulation (P1) with an example. The purpose is to see whether the supplier is better off offering trade credit to the buyer. Suppose c = Rs.3, A_b = Rs.40, A_s = Rs. 300, I_s =0.08 +0.06M, I_b = 0.12, I_c = I_p = 0.16, e = 2.5, θ = 0.10 and K = 400000. The solution maximizing (19) is found to be T = 0.1723years and v = Rs. 5.8454/unit. Equation (17) and (18) yield p = Rs. 9.1275/unit and M = .5506 years. With this policy the end demand D = 1589 units. The buyer's profit Π_b = Rs. 5569.989/year, and the seller's profit Π_s = Rs. 2206.267/year. The lot size in this solution is Q = 274 units.

Now, the sensitivity analysis of the seller's policy with respect to the parameters: the price elasticity of the end demand (e), the buyer's short term capital cost (Ic) and the constant rate of deterioration (θ) is presented in Table1 and Table2. In each analysis the base parameter values are as assumed in Example1 and only the parameter of interest is varied holding all other parameter constant.

It can be observed from table 1 that the optimal value of *M* increases with the decrease in the value of *e*. Therefore it would be economical for the supplier to offer more credit period when the end demand is less elastic. Moreover, items being deteriorating in nature therefore it is advisable to the buyer to order less but more frequently and take advantage of credit period to earn maximum profit.
From the Table 2, it is apparent that, for any value of θ, as the rate of financing increases optimum value of the length of credit period as well as the unit price charged by the seller increases, which implies that as the rate of financing increases it is advisable to the seller to offer large credit period and charge more.

8. CONCLUSIONS

In the present study, a model has been developed for deteriorating items to depict the relationship between seller - buyer in the presence of trade credit when the end demand is price sensitive. Since the decisions are mutually dependent for both i.e. the seller and the buyer, a leader-follower relation has been considered which provides a framework to the seller for coordinating his price and credit policy. Now, when the seller offers trade credit to the buyer, buyer needs to take into account the interaction between the unit price and the credit period before setting his policies. Moreover, results suggest that it would be economical for the seller to charge more and offer large credit period.

Table 2

For e = 2.5					
$\theta\downarrow$	**Ic→**	**0.10**	**0.12**	**0.14**	**0.16**
0.00	v	5.450279	5.556854	5.683627	5.836621
	p	9.211679	9.215937	9.181523	9.104008
	T	0.207261	0.196639	0.185934	0.174943
	M	0.087208	0.237567	0.392222	0.553795
	D	1553.149	1551.355	1565.933	1599.479
	Q	321.9073	305.0566	291.1603	279.8172
	Π_s	2295.277	2247.918	2227.485	2236.883
	Π_b	5529.849	5515.458	5535.930	5596.020
0.10	v	5.554849	5.656394	5.777000	5.922272
	p	9.455912	9.455473	9.415393	9.331086
	T	0.175887	0.169087	0.161737	0.153743
	M	0.067736	0.214657	0.365920	0.524098
	D	1454.794	1454.963	1470.496	1503.936
	Q	258.1425	248.1067	239.7681	233.0057
	Π_s	1997.403	1957.926	1942.242	1953.154
	Π_b	5275.142	5266.379	5290.805	5353.169
0.20	v	5.650751	5.748663	5.864429	6.003308
	p	9.680149	9.677474	9.633919	9.544813
	T	0.156744	0.151778	0.146143	0.139768
	M	0.050723	0.19444	0.342497	0.497431
	D	1372.003	1372.951	1388.521	1421.154
	Q	218.4589	211.5783	205.9168	201.4345
	Π_s	1746.719	1711.670	1698.133	1708.720
	Π_b	5057.282	5051.135	5077.055	5139.675
0.30	v	5.742104	5.837132	5.948813	6.082067
	p	9.893788	9.890148	9.844298	9.751437
	T	0.143609	0.139708	0.13511	0.129751
	M	0.035233	0.175924	0.320931	0.472763
	D	1299.133	1300.328	1315.522	1347.065
	Q	190.6441	185.5273	181.3912	178.2295
	Π_s	1528.039	1495.730	1483.093	1492.525
	Π_b	4862.803	4857.865	4884.102	4946.044

REFERENCES

ABAD, P. L. and C. K. JAGGI, (2003). "A Joint Approach for Setting Unit Price and The Length of The Credit Period For a Seller When End Demand is Price Sensitive**", Internal Journal of Production Economics 83**, 115-122.

CHANG, H.J. Aand DYE, C.Y. (2001). “An Inventory Model for Deteriorating Items With Partial Backlogging and Permissible Delay in Payments”, **International Journal of Systems Science** 32**,** 345-352.

CHAPMAN, C.B., WARD, S.C., COOPER, D.F. and PAGE, M.J. (1985). “Credit Policy and Inventory Control”, **Journal of the Operational Research Society** 35, 1055-1065.

CHUNG, K. H., (1989). “Inventory Control and Trade Credit Revisited”, **Journal of the Operational Research Society** 40 , 495-498.

CHUNG, K. J and C. K. H HUANG, (2000). “The Convexities of The Present Value Functions of All Future Cash Outflows in Inventory and Credit”, **Production Planning and Control.** 11, 133-140.

CHUNG, K. J., (1998). “A Theorem on Determination of Economic Order Quantity Under the Condition of Permissible Delay in Payment”, **Computer and Operations Research** 25 , 49-52.

CHUNG, K. J., (1999). “Bounds On The Optimum Order Quantity in DCF Analysis of The EOQ Model Under Trade Credit”, **Journal of Information and Operation Science** 20, 137-148.

CHUNG, K.J. (2000). “The Inventory Replenishment Policy for Deteriorating Items Under Permissible Delay in Payments”, **Opsearch 37**, 267-281.

COVERT, R.B. and PHILLIP, G.S. (1973). “An EOQ Model With Weibull Distribution Deterioration”, **AIIE Transactions** 5**,** 323-326.

FEWINGS, D. R. (1992). “A credit limit decision model for inventory floor planning and other extended trade credit arrangements”, **Decision Science** 23, 200-22.

GHARE, P.M. and SCHRADER, G.F. (1963). “A Model For Exponentially Decaying Inventory”, **Journal of Industrial Engineering** 14, 238-243.

GOYAL, S. K., (1985). “Economics Order Quantity Under Conditions of Permissible Delay in Payments”, **Journal of the Operational Research Society** 36, 335-338.

HALEY, C. W. and R. C. HIGGINS, (1973). “Inventory Policy and Trade Credit Financing”, **Management Science,** 20, 464-471.

HWANG, H. and S. W. SHINN, (1997). “Retailer's Pricing and Lot Sizing Policy for Exponentially Deteriorating Products Under the Condition of Permissible Delay in Payment”, **Computers Operations Research** 24, 539-547.

JAGGI, C. K. and S. P. AGGARWAL, (1994). “Credit Financing in Economic Ordering of Deteriorating Items”, **International Journal of Production Economics** 34, 151-155.

JAMAL, A. M. M., B. R. SARKER and S. WANG, (2000). “Optimal Payment Time for a Retailer Under Permitted Delay of Payment by the Wholesaler”, **International Journal of Production Economics** 66, 59-66.

KIN, J., H. HWANG and S. SHINN, (1995). “An Optimal Credit Policy to Increase Supplier's Profits With Price-Dependent Demand Functions”, **Production Planning & Control** 6 , 45-50.

MEHTA, D., (1968). “The Formulation of Credit Policy Models**”, Management Science** 15, B30-B50.

RAFAAT, F. (1991). “Survey of Literature on Continuously Deteriorating Inventory Models”, **Journal of the Operational Research Society** 42, 27-37.

SHAH, Y.K. and JAISWAL, M.C. (1977). “An Order level Inventory Model for a System with Constant Rate of Deterioration”, **Opsearch** 14**,** 174-184.

SHINN, S. W. and H. HWANG, (2003). “Optimal Pricing and Ordering Policies for Retailers Under Order Size-Dependent Delay in Payments”, **Computers and Operations Research** 30, 35-50.

SHINN, S. W., (1997). “Determining Optimal Retail Price and Lot Size Under Day-Terms Supplier Credit”, **Computers in Engineering** 33 **,** 717-720.

SHINN, S. W., H. HWANG and S. S. PARK, (1996). “Joint Price and Lot Size Determination Under Conditions of Permissible Delay in Payments and Quantity Discounts for Freight Cost”, **European Journal of Operational Research** 91**,** 528-542.

TENG, J. T., (2002**).** On the Economic Order Quantity Under Conditions of Permissible Delay in Payments, **Journal of the Operational Research Society** 53**,** 915-918.