INFERENCES ON THE FUNCTIONING OF ECONOMICS SYSTEMS

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ABSTRACT
The functioning of a finite economic system can be modeled through an adequate vector of dichotomic random variables. Each variable measures the functioning of a component. A linear probability model is used for modeling the effect of a set of external causes on the performance of the system. It' models, by a sequence of signals, the system's reliability. By observing the signals the monitoring of the system can be made and the shock is characterized by parameters. Predictors of them are developed. The approximate distributions of the predictors are derived. Portfolio analysis and consumer's selection of allocations illustrates the use of the model.

Key words: realibility, clustering, predictors

RESUMEN
El funcionamiento de un sistema económico finito puede ser modelado a través de un adecuado vector de variables aleatorias dicotómicas. Cada una mide el funcionamiento de un componente. Un modelo probabilístico lineal es usado para la modelación del efecto que, un conjunto de causas externas, tiene en el desenvolvimiento del sistema. Este modela mediante una sucesión de señales, la fiabilidad del sistema. Observando las señales el monitoreo del sistema es llevado a cabo y el choque es caracterizado por parámetros. Predictores de ellos son desarrollados. La distribución aproximada de ellos es derivada. El análisis de portafolios y de la selección de afijaciones de consumo son usados para ilustrar el uso del modelo propuesto.

Palabras clave: afijaciones de consumir, predictores.

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1. INTRODUCTION

A systemic analysis, of different problems, provides a theoretic frame that allows using common techniques for establishing if we can expect that certain models may function. A finite set of entities is clustered and the system works if all the clusters function. A certain function characterizes the functioning of the system. A set of weights can be computed for each evaluation and the reliability can be evaluated if the involved probabilities are known. The a priori information permits to use Bayes Theorem and some predictors are proposed.

Section 2 is devoted to the description of well-known economic models considering them as a system composed of clusters of entities. The independence of the components is assumed. Under a linear probability model a predictor is proposed. It is model unbiased. If a sufficiently large number of realizations of the system are observed a T-Student test statistic may be used.

Section 3 introduces the notion of shock. External causes influence in the functioning of the system. The independence is not longer valid. The reliability can be predicted when an adequate random experiment is performed. Again T-Student based inferences can be implemented.

Section 4 discusses some economical examples where this approach can be used.

2. SYSTEMS UNAFFECTED BY EXTERNAL CAUSES

Take a set of economic agents $\mathbf{N} = \{1,n\}$ and set of coalitions $\aleph = \{S\}$ such that $\cup_{S\in\aleph} S = \mathbf{N}$ and $S \cap S' = \phi$. The agent considers that his coalition performs in accordance with these objectives with a probability p_j (S). Therefore when the agents evaluate a coalition where they belong there exists a coalition a set of probabilities $P(S) = \{p_j = p_j(S) \in (0,1) | j \in S\}$. The evaluation of the agents seeks for establishing if S functions according to their aims. Each agent evaluates the dichotomic random variable

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$$X_j = \begin{cases} 1 & \text{if} \quad S \quad \text{functions} \\ 0 & \text{otherwie} \end{cases}$$

We will consider the usual problem of evaluating a strategy. The conditions of an economy vary and each coalition must adapt itself for surviving. The agents analyze the proposed strategies and evaluate each one. They must decide whether a proposed strategy allows the adequate performance of S or not. The agents coalitioned in S must reach a consensus on the correctness of the strategy for the future of the coalition. They agree whenever

$$X_S = \prod_{j \in S} X_j = 1$$

Let us consider some examples.

Example 2.1. The consumers are clustered in sets S. Each $j \in S$ proposes a consumption plan and its attainability be evaluated. $X_S = 1$ only if all the plans are attainable.

Example 2.2. Take S as a firm partitioned in divisions. Some changes are expected. The executives of each division evaluate the future of it under the new conditions. Different proposals are made to the firm's board. $X_S = 1$ when each division considers that the new politic fits with its aims.

Example 2.3. Different construction agencies present projects for constructing parts of a touristic resort. The investor considers them and clusters the partial projects. The corresponding agencies constitute a coalition S with a feasible whole project. When the investor considers that S provides an attainable project for the resort he fixes that $X_S = 1$.

An n-dimensional vector can represent the response of the agents of **N** with binary coordinates. Its n (S) agents represent each coalition. Take

$\boldsymbol{\chi} = \{X_{\underline{S}} \mid \underline{X_S} = (X_1, \ldots, Xn_{(S)})^T, S \in \aleph\}$.

A X_t is equal to one if the agent t considers that his coalition will function.

We will denote an economic system by ψ = {**N,** φ} where φ is a function that evaluates its functioning.

$$\varphi(\chi) = \begin{cases} 1 & \text{if} \quad \Psi \quad \text{functions} \\ 0 & \text{otherwise} \end{cases} .$$

The functioning of the economy depends on the independence of the agent's choice within each coalition S and of the parallelism of the coalitions. In Example 2.2 we may assume the independence of the division's decisions. The paralellism may be accepted if the firm can function without some divisions. That is the case when factories represent each division. A similar result holds for Example 2.3 because the investor is satisfied if at least one S fits his plans. In Example 2.1 if one coalition can not realize its consumption plan the corresponding consumers consider that the system is not functioning. Therefore if there does not exist an attainable plan the organization of the society fails.

We will consider only systems where the independence and the paralellism hypothesis hold for ψ. In practice each S has a certain importance for the system. It is measured by a weight $\delta(S) \in [0,1]$. Considering the weights the functioning of the economic system is measured by

$$\varphi(\chi) = \sum_{S \in \aleph} \delta(S) \prod_{j \in S} X_j$$

The weights of the coalitions are determined by solving the equation $\varphi(\chi)$=1. Note that ψ functions if t least a coalition functions and $\delta(S)$ = 1. That may be the case of Examples 2.1 and 2.2. $\varphi(\chi)$ is a random variable

because each agent assigns a probability to his decision on the functioning of the common strategy of his coalition. Its expectation depends of the set of subjective probabilities $\prod = \{P(S) = \{p_j \in (0,1) | j \in S\}\}$. For an assignment of the p_j 's it is:

$$E[\varphi(\chi)] = h(\Pi) = \sum_{S \in \aleph} \delta(S) \prod_{j \in S} p_j$$

The variance is given by

$$V[\varphi(\chi)] = \sum \delta^2(S) \prod p_j (1 - p_j)$$

The decision-maker designs an experiment and the random events w_1, ..., wm are generated independently from the corresponding probability space (Ω,σ,μ) where σ is a sigma algebra and μ is the probability measure that generates the set of subjective probabilities $\prod$. He observes a sample of estimates $\varphi(\chi(w_1)),...,\varphi(\chi(w_m))$ and computes an estimator of the multilinear function h $[\prod]$. A naive one is

$$\hat{h}(\Pi) = \frac{\sum_{t=1}^{m} \varphi(\chi(w_t))}{m}$$

Its error is

$$V(\hat{h}(\Pi)) = \frac{1}{m} \sum_{S \in \aleph} \delta^2(S) \prod_{j \in S} p_j (1 - p_j)$$

Hence the observer can estimate the reliability of the economic system using a sample of events. In practice he fixes a scenario and generates randomly some events. The decision rules are evaluated and a prediction of h $[\prod]$ is made. He can act as a Bayesian and he wishes to use the available data D. It is related with χ and is partially known. Some auxiliary information can be obtained and $Z = \{Z_1,..., Z_n\}$ is computed. The sampling design used for observing the w´s depends on Z: $d(s | Z)$. Gasymir-Natuig (1995) assumed that the agents fix Π and that it is perfectly known by all the involved agents. If the decision-maker assumes a prior distribution Q the behavior of Ψ can be analyzed using the Standard Bayes Theorem, see Gasymir-Natuig (1995). We will develop the inference using the Bayesian method proposed by Scott (1977). Then the use of Q yields

$$Q^D(Z|\chi(w)) = \begin{cases} 0 & \text{otherwise} \\ \dfrac{\int_\Omega d(s|Z)\partial^{\mu}(Q(\chi(w)|Z))}{d(s|Z)Q(\chi(w)|Z)} & \text{if} \quad \chi(w) \in \Omega \end{cases}$$

If Z is available at the data analysis stage $Q_D(\chi(w)| Z) \propto Q(\chi(w)| Z)$ the usual Bayesian procedures can be used for estimating

$$E(\hat{h}(\Pi)) = \int P(\varphi(\chi) = 1 | Z) \partial [Q_D(\chi(w) | Z)]$$

Note that the sampling design is unimportant for the inferences, as usual in Bayesian statistics. Hence the decision-maker can fix different scenarios, observe the results and develop the inferences using the prior Q_D.

The Bayesian approach will use the existence of a functional relation between $\chi(w)$ and Z for describing the prior through

$$E_\mu\left[\varphi(\chi(w))\middle|Z\right]=\beta Z_t+\varepsilon_t$$

and

$$Cov_\mu\left[\varphi(\chi(w)),\varphi(\chi(w'))\middle|Z\right]=\begin{cases}\sigma_t^2 & \text{if} \quad t=t' \\ 0 & \text{otherwise}\end{cases}$$

We can rewrite the linear probability model by fixing a coalition S that

$$(\varphi=(\chi_S)=\beta Z_S+\varepsilon_S \tag{2.1}$$

β is an unknown parameter and ε_t is an unobservable random variable with

$$E_\mu[\varepsilon_t]=0$$

and

$$Cov_\mu[\varepsilon_t,\varepsilon_{t'}]=\begin{cases}\sigma_S^2=\prod_{j\in S}p_j(1-p_j)=\prod_{j\in S}\sigma_j^2 & \text{if} \quad S=S' \\ 0 & \text{otherwise}\end{cases}$$

The data obtained through Z permits to predict

$$P((\varphi\chi_S)=1)=\beta Z_S$$

Therefore we deal with a Generalized Linear Model with binary random component and identity link function.

The use of $\varphi(\chi)$ as a predictor of h (Π) was suggested previously. As the sampling design is non-informative we can rely on the results of a reasonable experimental design for obtaining the required information. This allows the decision-maker to work with scenarios. The following proposition establishes that a T-Student approximation can be used for evaluating hypothesis on the reliability of the economic system for the set of subjective probabilities that characterizes a behavioral strategy of the economic agents.

Proposition 2.1. Suppose that a certain experimental design generates m random results of an economical system with a model structure $\psi = \sum_{S\in\aleph} S$. Take the function $\varphi_0(\chi(w))$ and the model given by (2.1). Assume that ψ is reliable when $h(\Pi) > \gamma$. The corresponding hypothesis may be tested by using the test statistics

$$T(\Pi)=\frac{\hat{h}(\Pi)-\gamma}{\sqrt{\frac{1}{m}\sum_{S\in\aleph}\delta^2(S)\prod_{j\in S}p_j(1-p_j)}}$$

$$\hat{h}(\Pi)=\frac{1}{m}\sum_{t=1}^{m}\varphi[\chi(w_t)]$$

if Π is known

$$t(\Pi)=\frac{\hat{h}(\Pi)-\gamma}{\sqrt{\frac{1}{m}\sum_{S\in\aleph}\delta^2(S)\prod_{j\in S}\hat{p}_j(1-\hat{p}_j)}}$$

$$\hat{p}_t=\frac{1}{m}\sum_{t=1}^{m}\varphi(\chi(w_t))$$

is the alternative statistics when it is only partially known by the decision-maker.

Proof:

$\varphi(\chi(w_t))$ is a Bernoulli random variable with weight $\lambda_t = 1/m$. If Π is unchanged during the experiment then

$$h(\Pi) = E_\mu[\hat{h}(\Pi)]$$

If we use the weights M_i

$$M_i = M \frac{\lambda}{\sum_{i=1}^{m} \lambda_i}$$

where

$$\lambda = \sum_{i=1}^{m} \lambda_i$$

$$M = \frac{\lambda^2}{\sum_{i=1}^{m} \lambda_i^2}$$

Then m is equal to the "equivalent sample size" defined by Pothoff et. al. (1992). From their result we have that for a sufficiently large m and a variable described by a superpopulation, t (Π) follows a T-Student distribution with m - 1 degrees of freedom approximately. Then the knowledge of the involved p_j's and their use in T (Π) justifies the normal approximation. From Bouza (1995) it follows that for the model (2.1) these results hold. □

3. SHOCK MODELS

A system can be affected in its performance by external causes. They will be denominated as "shocks". Formally they are described by:

Definition 3.1 A shock is an application

$$^{j}\nu(S) \mapsto \{0,1\}$$

that represents an exogenous cause that blocks the functioning of all $i \in S$.

Note that the influence of internal causes is not modeled by the shocks. In the previous examples we can identify as shocks for the corresponding systems:

Example 3.1. In the example 2.1 a shock is given by a change in the bonds market lead the firm to bankrupcity.

Example 3.2. In the example 2.2 the prices of the construction raw materials rise, news on a possible contamination of the shores near the touristic resort appear in the press and a competitor announces the opening of a similar resort.

Example 3.3. The wealth of the consumers for example 2.3 will be seriously reduced by devaluation of the currency, by a shortage of some products etc.

Following the ideas of Gasemyr-Natuig (1995) the decision-maker can identify a set of shocks for a coalition as $\kappa_S = \{^{j}\nu_S \mid j = 1,..., J\}$. Each agent is related with a set of shocks $E_i = \{B \mid i \text{ is shocked}\}$. If S does not function only when every agent is blocked the realization of $B^* \in \cap_{i\in S} E_i$ blocks S. When the blocking of only one of the components determines that S is not able to function it is sufficient that $B^*=E_i$ for at least one $i \in S$. We denote by E_S the set of shocks for S. The status of S is evaluated by

$$Y_B = \begin{cases} 1 & \text{if} \quad {}^j\nu_B \notin E_S \\ 0 & \text{otherwise} \end{cases}$$

where $E(Y_S) = \theta_S$.

Currently shocks can be considered as mutually independent. The involved agents obtain information from the environment in the form **a** signals. These signals suggest to him if a shock is expected. Hence he observes a sequence of **a** signals and fixes the value of X_i. Another point of view is to decide if S will function. Then Y_B is evaluated. The evaluation of it must be consistent with those of the X_i's. Take q_B as an unspecified marginal for Y_B. then

$$p_i = P(X_i = 1|w) = \prod_{B \notin E_i} q_B$$

Can be used in the inferences.

The sequence of signals suggests to the decision-maker which shocks may be expected. The marginals permit to compute a behavioral rule by computing the $p_{i'}$'s and using

$$X_i = \prod_{B \notin E_i} Y_B$$

The system can be characterized by the function

$$\varsigma : \{0,1\}^a \to \{0,1\}^n$$

that assigns to the sequence of **a** signals a 0-1 valued vector. The component i measures if the agent i decides the signals suggest that it will function or not. That is $\zeta(Y)=\chi$. Hence the information provided by Y permits to elicitate the performance of the system analyzing the possible shocks by computing

$$\eta(Y) = \varphi(\eta(Y)) = \sum_{S \in \aleph} \delta(S) \prod_{B \in E_S} Y_B$$

Then the system under shocks can be derived from ψ as $\{\aleph^*,\eta\} = \Gamma$, where. $\aleph^* = \cup_{j\in N} \{j\}$. Note that the components of Γ are independent because they are expressed in terms of Y.

The reliability of the functioning of the system is given by:

$$g(\Gamma) = \sum_{S \in \aleph} \delta(S) \prod_{B \notin E_S} \theta_B$$

The ideas discussed in the Section 2 holds for the shock-model because $E_\mu(\eta(Y)) = g(\Gamma)$. Therefore is easily obtained that

$$V_\mu(\eta(Y)) = \Sigma_{S\in\aleph}\, \delta^2(S)\, \Pi_{B\notin E_S}\, \theta_B (1-\theta_B)$$

is estimated by

$$\hat{V}_\mu(\eta(Y)) = \Sigma_{S\in\aleph}\, \delta^2(S)\, \Pi_{B\notin E_S}\, \hat{\theta}_B (1-\theta_B)$$

and T-Student based confidence intervals for $g(\Gamma)$ can be used by estimating θ_B by

$$\hat{\theta}_B = \frac{1}{m} \sum_{t=1}^{m} Y_{B(t)}$$

When a sample of size m is selected.

4. SOME ADDITIONAL EXAMPLES

The described systems can be used for modeling different well-known problems. Some of them are given below.

Example 4.1. Portfolio analysis.

Take a market of bonds with fixed revenues at the times t = 0,1,...., T., the set of bonds of the decision-maker and the exchange rats $\{a_{i\,(j)}, j = 1..., T\}$. The $a_{i\,(j)}$´s are known non random variables variables variables variables variables variables variables variables variables variables at t=0. The market's matrix is given by

$$M = \begin{bmatrix} a_{1(1)} \cdots a_{1(n)} \\ \vdots \quad \vdots \quad \vdots \\ a_{T(1)} \cdots a_{T(n)} \end{bmatrix}$$

Denote the vector price at t = 0 by $C^T = (c^{(1)},..., c^{(n)})$. The columns of M are the exchanges rates related with the bonds. Under the hypothesis of non-arbitrage and completeness the prices can be expressed by

$$c^{(k)} = \sum_{t=1}^{T} B(0,T)a_{t(k)}$$

where B(0,t) is the actualization factor time at time t, see Frachet (1996). Then it is possible to reconstruct the zero coupons as a linear combination of the existent market-unitary bonds. The decision-maker evaluates his portfolio by using $a_{t\,(i)}$ and he fixes that $X_i = 1$ when $c_{(i)} > G_{(i)}$. The realibility of the portfolio can be predicted under different scenarios characterized by a set of χ's. The portfolio "functions" for a scenario χ when

$\eta(\chi) = 1$, that is when he considers that revenues are sufficiently high.

Example 4.2. Selection of an allocation by a consumer.

Take a consumer with a positive welfare w. He should select a consumption vector c from the set

β = Argmax {u(c)}

Subject to $\{c \in \Re^n_+ \mid \sum_{i=1}^n p_i c_i \leq w\} = \Delta$

when the price system is $p^T = (p_1,...,p_n)$ and the utility function is u(c). The consumer can select an allocation c by evaluating $F:\beta \rightarrow \{0,1\}^n$. Then the j-th good will be consumed if $X_j = 1$. The action of the consumer is feasible if the selected c belongs to the demand set Δ. The goods may be placed in different sets. They generally identify a differential consumption pattern. Increment in the prices is an external cause that affects the system. Some signals as the dependence of different goods of some raw materials, a common politic of a set of productors, etc give signals to the consumer.

The consumer fixes $X_i = \Pi_{B \in E_i} Y_B$ and ellicitates q_B from the information (signals) of the market. If he knows the value of all the Y_B's he can predict the reliability of his consumption strategies.

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